TURBULENCE ENERGY BALANCE IN AXISYMMETRIC WAKES BEHIND BODIES OF DIVERSE SHAPES

V. I. Bukreev, V. A. Kostomakha,
and Yu. M. Lytkin

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The structure of turbulence in axisymmetric wakes behind a sphere and behind a body of revolution with an 8 : 1 span, around which an incompressible fluid streams at a constant velocity U_∞ is investigated experimentally. The tests were performed in a low-turbulence wind tunnel at a Reynolds number $Re = U_\infty D/\nu = 10^4$ (D is the diameter of the middle section) at sufficiently long ranges from the streamlined bodies, where the flow mode in the wake becomes completely self-similar.

It has been shown in [1] that the turbulence characteristics in a self-similar wake, determined by the large-scale motion components, depend not only on the body drag and the free-stream velocity but also directly on the body shape. The influence of the body shape on the various components of the turbulence energy balance in the wake is illustrated herein.

1. A thermoanemometer apparatus of the firm "DISA Elektronik" and a spectrum analyzer of the form "Brulle and Kier" were used in the research. Additional information about the test conditions is contained in [1]. The experimental results obtained permit estimation of the various members of the turbulence energy balance equation in a wake

$$U_\infty \frac{\partial e}{\partial x} = -\frac{1}{r}\frac{\partial}{\partial r} r \left\langle v\left(\frac{u^2 + v^2 + w^2}{2} + \frac{p}{\rho}\right)\right\rangle + \langle uv\rangle \frac{\partial U_1}{\partial r} - \varepsilon \tag{1.1}$$

Here a cylindrical coordinate system x, r, φ, coupled to the body is used (the x axis coincides with the wake axis of symmetry and is directed downstream, the origin is at the body trailing edge), $U_1 = U_\infty - U$ is the deficit of the average longitudinal velocity U in the wake; u, v, w are the axial, radial, and tangential pulsating velocity components; $e = 1/2\,(\langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle)$ is the turbulence energy per unit mass of fluid; ε is the rate of dissipation of this energy into heat, p is the pressure; ρ is the fluid density; and the angular parentheses denote averaging.

The experimental results presented below refer to the section x/D = 100. As has been shown in [1], the flow in the wake is completely self-similar at such a distance from the bodies under consideration. This means that the statistical characteristics of turbulence, which are determined by the large-scale motion components can be represented as

$$U_1 = U_c f_1(r/l_c), \quad \langle u^2\rangle = U_c^2 f_{11}(r/l_c), \quad -\langle uv\rangle = U_c^2 f_{12}(r/l_c) \tag{1.2}$$

etc. The characteristic scales of the velocity U_c and the length l_c in Eq. (1.2) depend only on the coordinate x and are given by

$$U_c = U_\infty \left[(x - x_0)/\sqrt{c_x S}\right]^{-2/3}, \quad l_c = \sqrt{c_x S}\left[(x - x_0)/\sqrt{c_x S}\right]^{1/3}$$

where x_0 = const is the virtual origin of the self-similar wake, and c_x is the drag coefficient determined by using the characteristic body area S. The quantity x_0 in these investigations was determined by the results of measuring U_1 on the wake axis and turned out to be zero in both the cases under consideration.

It follows from Eq. (1.1) that the dimensionless specific dissipation $\varepsilon l_c / U_c^3$ depends only on r/l_c.

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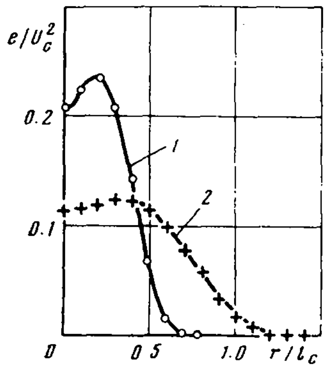

Fig. 1

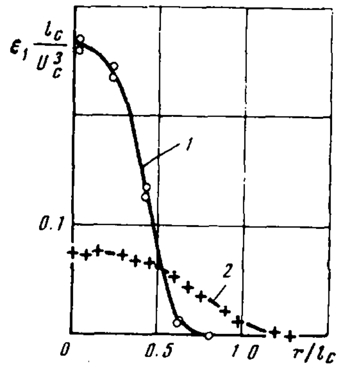

Fig. 2

2. Presented in Fig. 1 are self-similar turbulence energy profiles in the wakes behind an elongated body (curve 1) and behind a sphere (curve 2). The left side of Eq. (1.1), describing the convective transport of turbulence energy, can be found from this profile. The first member in the right side of Eq. (1.1) characterizes the turbulent diffusion of this energy. Since the correlation $\langle vp \rangle$ therein cannot be measured successfully, the diffusion member of Eq. (1.1) is usually estimated in experimental investigations as the difference between the remaining terms. The generation of turbulence energy can be computed from the profiles of U_1 and $\langle uv \rangle$ presented in [1].

As a rule the approximation

$$\varepsilon \approx \varepsilon_1 = 15\nu \langle (\partial u / \partial x)^2 \rangle \tag{2.1}$$

is used in an experimental determination of the specific dissipation of the turbulence energy $\langle (\partial u/\partial x)^2 \rangle$ by assuming the flow to be locally isotropic. The variance E(k) was found herein by the results of measuring the spectrum densities E(k) of the fluctuations of the u-components of the velocity from the formula

$$\left\langle \left(\frac{\partial u}{\partial x} \right)^2 \right\rangle = \int_0^\infty E(k)\, k^2\, dk$$

where $k = 2\pi f/U$ is the wave number (f is the fluctuation frequency in Hertz). The distribution of the quantity $\varepsilon_1 = 15\nu \langle (\partial u / \partial x)^2 \rangle$ over the transverse section of the wake is presented in Fig. 2 (curve 1 corresponds to the elongated body, and curve 2 to the sphere).

The accuracy of the approximation (2.1) can be estimated by means of the known integral relation [2]

$$U_\infty \frac{d}{dx} \int_0^\infty \left(\frac{U_1^2}{2} + e \right) r dr = - \int_0^\infty \varepsilon r dr$$

which becomes when the self-similarity of the U_1, e, and ε profiles are taken into account

$$\frac{1}{U_c^2 l_c^2} \int_0^\infty \left(\frac{U_1^2}{2} + e \right) r dr = \frac{3}{2 U_c^3 l_c} \int_0^\infty \varepsilon r dr \tag{2.2}$$

The right side of Eq. (2.2), computed from the experimental profiles of $\langle (\partial u/\partial x)^2 \rangle$ turned out to be less than the left side by approximately 30% in both the cases considered. This discrepancy is small. The value of the integral of the dissipative term turned out to be still more reduced in [3] (plane jet), [4] (wake behind a circular disk), and [5] (wake behind a sphere), in which the approximation (2.1) was used.

The left side of Eq. (2.2), computed from experimental results, has the same value 0.05 in the wake behind the sphere and in the wake behind the elongated body of revolution. It hence follows, in particular, that despite the essential difference in the relationship between $U_1^2/2$ and e in the wakes behind the considered bodies of different shape, the dimensionless kinetic energy flux

$$\frac{1}{U_\infty^3 c_x S} 2\pi U_\infty \int_0^\infty \left(\frac{U_1^2}{2} + e \right) r dr$$

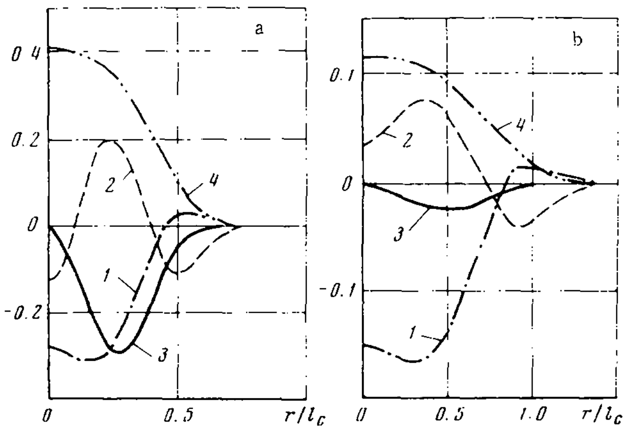

Fig. 3

in a self-similar axisymmetric wake is the universal function $(x - x_0)/\sqrt{c_x S}$, independently of the body shape.

3. Shown in Fig. 3 is the distribution of various members of Eq. (1.1) multiplied by l_c/U_c^3 over the wake cross-section (1 is convection, 2 is diffusion, 3 is generation, 4 is dissipation, *a* is the elongated body, and b is the sphere). Here, as in [3], the experimental values $\varepsilon_1 = 15\nu\langle(\partial u/\partial x)^2\rangle$, multiplied by a constant correction factor in order to satisfy Eq. (2.2), were used as the dissipation ε. Such a method of introducing the correction is not rigorous, but it assures that the integral of the diffusion member in Eq. (2.2) over the wake transverse section would be zero.

It is seen from Fig. 3 that the convective turbulent energy transport in axisymmetric wakes turns out to be substantial, and in contrast to diverse near-wall turbulent flows the zone in which the members responsible for turbulent energy generation and dissipation would be considerably greater in absolute value than the remaining terms in Eq. (1.1) is not isolated successfully. Let us note the different relative contribution of the member $\langle uv\rangle\,\partial U_1/\partial r$ to the turbulent energy balance in the cases of the elongated body and the sphere. The ratio between the energy fluxes

$$J = 2\pi U_\infty \int_0^\infty \frac{U_1^2}{2} r\,dr / 2\pi U_\infty \int_0^\infty e r\,dr$$

which is a constant in the self-similar wake, can be compared. This quantity depends essentially on the body shape and equals 1.5 and 0.2 in the wakes behind an elongated body and the sphere, respectively.

The relatively small value of J in the wake behind the sphere does not mean that this flow is almost shear-free in structure. The maximum value $|R|_{max}$ of the correlation coefficient $R = \langle uv\rangle\sqrt{\langle av\rangle^2\langle v^2\rangle}$ in the transverse section of a self-similar wake behind a sphere turned out to be 0.50, i.e., it is somewhat greater than the value $|R|_{max} = 0.42-0.45$, characteristic for turbulent shear flows in tubes, boundary layers, etc. In the wake behind an elongated body of revolution $|R|_{max} = 0.53$.

LITERATURE CITED

1. V. I. Bukreev, O. F. Vasil'ev, and Yu. M. Lytkin, "On the influence of body shape on the characteristics of a self-similar axisymmetric wake," Dokl. Akad. Nauk SSSR, 207, No. 4, 804-897 (1972).
2. A. A. Townsend, Structure of Turbulent Shear Flow, Cambridge Univ. Press.
3. L. J. S. Bradbury, "The structure of a self-preserving turbulent plane jet," J. Fluid Mech., 23, Pt. 1, 31-64 (1965).
4. N. H. Z. Huan and L. W. Baldwin, "Damping of turbulence in axisymmetric wakes," Trans. ASME, Ser. D, Theor. Princ. Eng'ng Comp., 88, No. 1, 81-90 (1966).
5. M. S. Uberoi and P. Freymuth, "Turbulent energy balance and spectra of an axisymmetric wake," Phys. Fluids, 13, No. 9, 2205-2210 (1970).